This paper investigates the “symplectic Khovanov homology” $\text{Kh}_{\text{symp}}$ introduced by P. Seidel and I. Smith [Duke Math. J. 134 (2006), no. 3, 453–514; MR2254624]. $\text{Kh}_{\text{symp}}$ is an invariant of knots in $S^3$; it is conjectured to be isomorphic to M. G. Khovanov’s Jones polynomial homology [Duke Math. J. 101 (2000), no. 3, 359–426; MR1740682]. Roughly speaking, it is constructed as follows. For each $n > 0$, Seidel and Smith described an open symplectic manifold $Y_n$, equipped with an action of the braid group $\text{Br}_n$, and containing a certain Lagrangian submanifold $L_n$. If the knot $K$ is the closure of a braid $b \in \text{Br}_n$, its symplectic Floer homology is defined to be the Lagrangian Floer homology of the pair $(L_n, b(L_n))$ in $Y_n$.

The paper under review gives a very nice, concrete description of $Y_n$ and $L_n$. The author shows that $Y_n$ can be identified with an open subset of the Hilbert scheme $\text{Hilb}^n(S)$, where $S$ is an affine surface in $C^3$. $S$ contains $n$ Lagrangian spheres $\Sigma_1, \ldots, \Sigma_n$, and the Lagrangian $L_n$ can be replaced with the image of the product $\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n$ in $\text{Hilb}^n(S)$. Using this identification, the author explicitly describes the intersection points $L_n \cap b(L_n)$ in terms of a bridge diagram for $K$. He shows that they are in bijective correspondence with the intersections which appear in S. J. Bigelow’s “homological definition of the Jones polynomial” [in Invariants of knots and 3-manifolds (Kyoto, 2001), 29–41 (electronic), Geom. Topol. Monogr. 4, Geom. Topol. Publ., Coventry, 2002; MR2002601], thus lending support to the conjecture that $\text{Kh}_{\text{symp}}$ is isomorphic to Khovanov homology.

The definition of $\text{Kh}_{\text{symp}}$ considered here is remarkably similar to that of Heegaard Floer homology [P. S. Ozsváth and Z. Szabó, Ann. of Math. (2) 159 (2004), no. 3, 1027–1158; MR2113019]. The author pursues this relationship further, showing that there is a natural correspondence between the intersection points $L_n \cap b(L_n)$ the generators of the Heegaard Floer complex $\text{CF}(\Sigma(K)\# S^1 \times S^2)$, where $\Sigma(K)$ denotes the double cover of $S^3$ branched along $K$. This result is of particular interest in light of the work of Ozsváth and Szabó [Adv. Math. 194 (2005), no. 1, 1–33; MR2141852] which implies that there is a spectral sequence starting at $\text{HF}(\Sigma(Y)\# S^1 \times S^2; \mathbb{Z}/2)$ and converging to $\text{Kh}(K; \mathbb{Z}/2)$.

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32. ———, Symplectic geometry of the adjoint quotient, II, lecture, MSRI Workshop on Symplectic Geometry and Mathematical Physics, Mathematical Sciences and Research Institute, University of California, Berkeley, March 2004. 315, 316, 357
34. I. SMITH, Symplectic geometry of the adjoint quotient, I, lecture, MSRI Workshop on Symplectic Geometry and Mathematical Physics, Mathematical Sciences and Research Institute, University of California, Berkeley, March 2004. 315, 316, 357
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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.