The existence of rank-one mixing transformations was proved by D. S. Ornstein in 1974 [Ergodic theory, randomness, and dynamical systems, Yale Univ. Press, New Haven, Conn., 1974; MR0447525]. His result is based on probabilistic cutting and stacking constructions with random spacers. Explicit cutting and stacking constructions (staircase transformations) leading to rank-one mixing transformations were done by T. M. Adams [Proc. Amer. Math. Soc. 126 (1998), no. 3, 739–744; MR1443143].

The author gives a differentiable realisation of a rank-one and mixing flow. More precisely, he introduces an uncountable dense subset \( Y \subset \mathbb{R}^2 \) such that for any \( (\alpha, \alpha') \in Y \) there exists a strictly positive real function \( \varphi \) defined on \( T^2 \) (two-dimensional torus) of class \( C^1 \) such that the special flow built over \( R_{\alpha, \alpha'} \) (the rotation on \( T^2 \) by \( (\alpha, \alpha') \)) with the ceiling function \( \varphi \) is rank one and mixing with respect to its unique invariant measure.

To define the special flow \( T^l_{T, \varphi} \) over a dynamical system \((M, T, \mu)\) and under the ceiling function \( \varphi \in L^1(M, \mu), \varphi \geq c > 0 \), consider the subset \( M_{\varphi} \subset M \times \mathbb{R} \), where \( M_{\varphi} = \{(z, s) \in M \times \mathbb{R}; 0 \leq s \leq \varphi(z)\} \). The special flow acts on the manifold \( M_{T, \varphi} \) obtained from \( M_{\varphi} \) by identifying pairs \((z, \varphi(z))\) and \((Tz, 0)\). It preserves the normalised product measure on \( M_{T, \varphi} \), i.e., the product of the measure \( \mu \) with the Lebesgue measure on the fibres divided by \( \int_M \varphi(z) \mu(dz) \).

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.