Let \((M, \mu)\) be a Lebesgue space, \(T\) an automorphism, and \(f(n)\) a sequence of positive reals tending to zero. \(T\) is said to allow approximation by periodic transformations with speed \(f(n)\) \([\text{speed} o(f(n))]\) if there is a sequence of finite partitions \(\zeta_n = \{C_{nk}\} (1 \leq k \leq q_n)\), \(\zeta_n \to \varepsilon\) \((\text{the trivial partition})\), and periodic automorphisms \(S_n\) of \((M, \mu)\) of period \(q_n\), with \(S_n\zeta_n = \zeta_n\), such that \(\sum_{k=1}^{q_n} \mu(TC_{nk} \Delta S_nC_{nk}^{-1}) < f(q_n)\) \([= o(f(q_n))]\).

The first paper is concerned with speed \(o(f(q_n))\). If \(f(q_n) = 1/\ln^2 q_n\), then the entropy \(h(T) = 0\). If \(f(q_n) = 1/q_n\), then \(T\) is ergodic but not mixing, \(U_{T_q} \to E\) strongly, and the maximal spectral type of \(U_T\) is singular. These considerations are applied to the construction of ergodic automorphisms and flows. One such construction is as follows.

Let \((M, \mu)\) be the circle with Lebesgue measure. Let \(M' = M \times \mathbb{Z}_2\), \(T'(x, j) = (x + \alpha, n(x)j)\), where \(\alpha\) is irrational, \(n(x) = -1\) on \([0, \gamma)\) and \(n(x) = 1\) on \((\gamma, 1]\). If \(\alpha\) and \(\gamma\) satisfy certain conditions related to approximation by rationals, and \(H_{-1}\) is the subspace of \(L^2(M')\) defined by \(f(x, 1) = -f(x, -1)\), then \(U_{T'}\) has continuous spectrum on \(H_{-1}\). If \(\sigma\) is the maximal spectral type of \(U_{T'}\) then \(\sigma \perp \sigma^*\sigma\).

In the second paper, it is shown that any automorphism may be approximated with speed \(a_n/\ln n\), where \(a_1 \leq a_2 \leq \cdots\) and \(a_n \to \infty\). If \(c(T) = \inf \{c: T\text{ allows approximation with speed } c/\ln n\}\), then, if \(T\) is ergodic, \(h(T) \leq c(T) \leq 2h(T)\). If \(T\) is ergodic, it can be approximated with speed \(o(1/\ln n)\) if and only if \(h(T) = 0\).

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