Hausdorff dimension of the set of nonergodic directions. (English summary)
With an appendix by M. Boshernitzan.

The paper concerns billiards in rational polygons whose angles are rational multiples of \( \pi \). For this class of polygons, the billiard system is well studied and a wealth of results are known; see [H. A. Masur and S. L. Tabachnikov, in Handbook of dynamical systems, Vol. 1A, 1015–1089, North-Holland, Amsterdam, 2002; MR1928530] for a survey.

One of these results, due to S. P. Kerckhoff, H. A. Masur and J. Smillie [Ann. of Math. (2) 124 (1986), no. 2, 293–311; MR0855297], is that the set of directions in which the billiard flow is not ergodic has zero measure. In fact, H. A. Masur [Duke Math. J. 66 (1992), no. 3, 387–442; MR1167101] proved later that the set of non-ergodic directions has at most Hausdorff dimension 1/2.

In the present paper, the latter bound is proved to be sharp. The relevant billiard table is a \( 2 \times 1 \) rectangle with a slit parallel to the shorter side passing through the middle of a longer side and of length \( 1 - \lambda \). The main result is that if \( \lambda \) is a Diophantine number, then the Hausdorff dimension of the set of non-ergodic directions is equal to 1/2. The method of proof is number-theoretical: it consists of estimating from below the number of primitive lattice points in certain subsets in the plane.

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References

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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