Rational billiards and flat structures.


This chapter surveys the theory of billiards on Euclidean polygons whose angles are rational multiples of $\pi$. To each such polygon, by unfolding the billiard trajectories one can associate a compact Riemann surface with a flat structure (in fact a translation structure) having conical singularities with cone angles which are multiples of $2\pi$. The billiard trajectories in the polygon correspond to geodesics (straight lines) on the surface—at least when they avoid the cone points.

This construction puts the billiard problem into the larger context of Teichmüller theory on Riemann surfaces. In particular many deep results are obtained by studying the ergodicity of the natural $\text{SL}(2,\mathbb{R})$-action on the corresponding moduli space.

A number of the most important basic results are presented, such as ergodicity, counting periodic orbits, Veech surfaces, and results on interval exchange transformations.

{For the collection containing this paper see MR1928517}

Richard Kenyon

© Copyright American Mathematical Society 2018