The authors investigate joinings between two strongly irreducible totally non-symplectic (TNS) Anosov $\mathbb{Z}^k$-actions $\alpha_1$ and $\alpha_2$ by automorphisms of $\mathbb{T}^{m_1}$ and $\mathbb{T}^{m_2}$ (where TNS implies that $k \geq 2$). A description of joinings is currently out of reach, since every invariant measure for the action $\alpha_1$ gives rise to a self-joining between $\alpha_1$ and $\alpha_2$. However, the current paper proves very strong implications if $\alpha_1$ and $\alpha_2$ admit a nontrivial joining. This extends earlier work on rigidity of invariant measures and other rigidity properties for those and similar higher rank abelian actions [see A. B. Katok and R. J. Spatzier, Ergodic Theory Dynam. Systems 16 (1996), no. 4, 751–778; MR1406432; erratum; MR1619571; B. Kalinin and A. B. Katok, in Smooth ergodic theory and its applications (Seattle, WA, 1999), 593–637, Proc. Sympos. Pure Math., 69, Amer. Math. Soc., Providence, RI, 2001; MR1858547; A. Katok, S. Katok and K. Schmidt, “Rigidity of measurable structure for $\mathbb{Z}^d$-actions by automorphisms of a torus,” preprint, arXiv.org/abs/math/0003032].

For the statement of the main theorem, suppose $\mu$ is a nontrivial joining on $\mathbb{T}^{m_1} \times \mathbb{T}^{m_2}$ between $\alpha_1$ and $\alpha_2$, that is, a measure which is invariant under the $\mathbb{Z}^k$-action $\alpha_1 \times \alpha_2$ and projects to the Lebesgue measures on $\mathbb{T}^{m_i}$ for $i = 1, 2$ but is not the Lebesgue measure on the product. Then the dimensions of the tori agree ($m_1 = m_2$), and there exists a subgroup $\Gamma \subset \mathbb{Z}^k$ of finite index such that $\alpha_1$ and $\alpha_2$ restricted to $\Gamma$ are isomorphic over $\mathbb{Q}$, so they are finite-to-one factors of each other.

This is the strongest possible implication for the relation between the two actions because of the following construction. Suppose $\alpha_1$ and $\alpha_2$ are two actions as above satisfying that their restrictions to a finite-index subgroup $\Gamma \subset \mathbb{Z}^k$ are conjugate over $\mathbb{Q}$. Those restrictions are factors of each other. Let $\mu'$ be a joining induced by one of those factor maps. The orbit average $\mu$ of $\mu'$ under the full action $\alpha_1 \times \alpha_2$ is a nontrivial joining between $\alpha_1$ and $\alpha_2$.

The proof of the above theorem relies on an extension of earlier methods, and an estimate for entropy—a relative version of the Ruelle inequality.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.