Let $G$ be a compact connected Lie group and $K$ a closed connected subgroup. Denote by $X$ a symplectic manifold on which $G$ acts in a Hamiltonian fashion. Let $p: X \to \mathfrak{g}^*$, where $\mathfrak{g}$ is the Lie algebra of $G$, be the moment mapping. The functions of type $h \circ p$, for $h: \mathfrak{g}^* \to \mathbb{R}$, are called collective. Such $h \circ p$ are integrals for any flow on $X$ with $G$-invariant Hamiltonian (Noether’s theorem). A completely integrable system consisting of $(\dim X)/2$ independent real-analytic functions of this type commuting with respect to the Poisson bracket is called a collective completely integrable system. For example, all symmetric spaces $G/K$ admit a collective completely integrable system on the phase space $T^*(G/K)$. Moreover, on $T^*(G/K)$ there exists a collective completely integrable system if and only if the subgroup $K$ of $G$ is spherical. In this case, we call $(G, K)$ a spherical pair.

Let $N_{\text{max}}$ be the maximal number of independent real-analytic commuting functions on $X = T^*(G/K)$ of type $h \circ p$. If $N_{\text{max}} = (\dim X/2) - 1$ we call the corresponding system of functions an almost collective completely integrable system and the subgroup $K$ an almost spherical subgroup of $G$. In this case, the pair $(G, K)$ is called an almost spherical pair.

In this paper, the authors enumerate all almost spherical pairs $(G, K)$ for a simple compact Lie group $G$.

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