The main aim of the book is to provide a rather comprehensive study of optimal control problems within the framework of infinite-dimensional theory with particular emphasis on dynamics governed by partial differential equations. A short description of the content of the book is given below.

Chapter 1 deals with questions related to existence and uniqueness of solutions to optimal control problems. General results in an abstract setting are followed by more concrete examples pertaining to both parabolic and hyperbolic dynamics.

Optimal solutions are characterized by a system of optimality in Chapter 2. Both necessary and sufficient conditions for optimality are discussed in the book. The results presented here deal mostly with parabolic equations (linear and nonlinear) and with various types of controls which include distributed controls, boundary controls and controls in initial conditions.

Solvability of boundary value problems associated with “ill-posed” equations is the main theme of Chapter 3. Here, the author concentrates on three types of problems: (i) Cauchy data associated with elliptic operators, (ii) backward parabolic equations, and (iii) Navier-Stokes equations in the three-dimensional case with very special forcing terms. The main aim is to establish global existence and uniqueness of solutions for a dense set of data. This type of results is essential in studying, later in the book, corresponding control problems associated with ill-posed dynamics.

Chapter 4 deals with rigid control and non-coercive functional cost. After presenting rather general results at an abstract level, the author provides a detailed study for a concrete example of work minimization in accelerating still fluid to a prescribed velocity. Necessary and sufficient optimality conditions are derived and discussed for this problem.

Boundary control of the Navier-Stokes equation is the main topic of Chapter 5. Existence and uniqueness of optimal solutions is thoroughly discussed. An optimal solution is then characterized by a system of optimality. Rigorous justification of various limiting arguments is achieved with the help of refined trace regularity obtained for the optimal solutions. In fact, trace theory plays a critical role in this analysis.

Chapter 6 discusses optimal control theory associated with ill-posed elliptic problems, where the results of Chapter 4 are employed. Motivation for studying this type of problem comes from applications in geophysics.

Local exact controllability for two-dimensional flows of an incompressible viscous fluid (Navier-Stokes equations) is discussed in Chapter 7. The problem is tackled, as usual, by reducing the issue to controllability of linearized dynamics. The main technical tools here are Carleman’s estimates, which are derived at the end of the chapter.

The results provided in this book are interesting and focused on parabolic-like control problems with particular emphasis on Navier-Stokes equations. The proofs are complete
and accessible to non-specialists. Many results presented in the book are relatively recent contributions to the field which are based on research work done by the author himself or in collaboration with O. Yu. Emanuilov (Imanuvilov).

The book provides a very good source of information for researchers in the area of control theory for partial differential equations. It also can be used as a graduate level textbook for students entering the field. Some background in functional analysis, function space theory and PDEs is needed.

{The Russian original has appeared [Nauchnaya Kniga, Novosibirsk, 1999; Zbl 938.93003].}

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