The truncated complex $K$-moment problem. (English summary)


Let $\gamma_{pq},\ 0 \leq p,q \leq d,$ be a finite Hermitian matrix of complex numbers satisfying $\gamma_{00} > 0$. The necessary and sufficient conditions for the existence of a positive Borel measure $\mu$ in the complex plane, so that $\gamma_{pq} = \int z^p \overline{z}^q d\mu(z),$ are discussed, in conjunction with the requirement that $\text{supp}\{\mu\}$ is contained in a given real algebraic curve. If a solution $\mu$ exists, then by elementary convexity theory one proves that an atomic solution exists, too (Tchakaloff’s theorem).

This truncated moment problem is equivalent to the construction of Gaussian cubature of a given order for a prescribed measure [cf. I. P. Mysovskikh, *Interpolation cubature formulas* (Russian), “Nauka”, Moscow, 1981; MR0656522]. In either approach, very little progress was made in the last decades. Also, the existing results cover only very special measures and very low degrees.

The main feature of Curto and Fialkow’s paper is its algorithmic aspect, which by simple matricial computations makes possible the identification of the nodes and weights of a minimal, atomic representing measure for the given sequence $\gamma.$ Unfortunately this does not solve the truncated moment problem in its entirety, because a couple of strong degeneration conditions have to be met.

The work is addressed to operator theorists, but it may well be studied and exploited by experts in numerical integration.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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