Katok, A. [Katok, Anatole B.] (1-PAS); Spatzier, R. J. [Spatzier, Ralf J.] (1-MI)
Nonstationary normal forms and rigidity of group actions. (English summary)

The authors announce new results on $C^\infty$-rigidity of homogeneous Anosov actions of higher rank. First they develop a “nonstationary” generalization of the classical theory of normal forms for local contractions. More specifically, let $X$ be a compact metric space, $V$ a vector bundle over $X$, $f: X \to X$ a homeomorphism, and $F: V \to V$ a continuous extension of $f$ which is smooth along the fibers. Suppose that $F$ is a contraction with narrow band spectrum. Then $F$ is equivalent to a polynomial map of subresonance type. Moreover, if $G$ is a continuous, fiberwise smooth extension of a homeomorphism $g: X \to X$, and $G$ commutes with $F$, then $G$ is also a polynomial map of subresonance type. This shows that a local action of an abelian group by extensions which contain a contraction with narrow band spectrum can be simultaneously reduced to a normal form.

Now let $M$ be a compact manifold, and $\rho$ a $C^\infty$ action of a finitely generated discrete group $\Gamma$ on $M$. Then $\rho$ is said to be $C^\infty$ locally rigid if any $C^\infty$ action $\tilde{\rho}$ of $\Gamma$ on $M$ that is $C^1$ close to $\rho$ on a set of generators is $C^\infty$ conjugate to $\rho$. The action is Anosov if $\rho(\Gamma)$ contains an Anosov element. The result is as follows. Any algebraic Anosov action of $\mathbb{Z}^k$, $k \geq 2$, on an infranilmanifold is $C^\infty$ locally rigid provided all nontrivial elements of $\mathbb{Z}^k$ are ergodic and semisimple. The semisimplicity condition is technical and hopefully may be omitted. This extends the previous result of the authors [Inst. Hautes Études Sci. Publ. Math. No. 79 (1994), 131–156; MR1307298] for Anosov actions of $\mathbb{Z}^k$ on tori, and is a corollary of a general result for Anosov $\mathbb{R}^k$-actions based on the theory of normal forms.

From this the authors deduce the following. Let $\Gamma$ be an irreducible lattice in a semisimple Lie group $G$ with no real rank 1 factors. Then any algebraic Anosov $\Gamma$-action on a nilmanifold is $C^\infty$ locally rigid. This uses Zimmer’s cocycle superrigidity theorem and generalizes an earlier result of Katok, J. Lewis and R. Zimmer [Topology 35 (1996), no. 1, 27–38; MR1367273].

Finally, let $\Gamma$ be a cocompact irreducible lattice in a semisimple Lie group $G$ with finite center, of real rank at least 2, and without compact factors. Let $P$ be a parabolic subgroup of $G$. Then the left action of $\Gamma$ on $G/P$ is $C^\infty$ locally rigid. This kind of projective action for $G = \text{SL}(2, \mathbb{R})$ was first studied by É. Ghys [Inst. Hautes Études Sci. Publ. Math. No. 78 (1993), 163–185 (1994); MR1259430].

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