Self-avoiding random walks in five or more dimensions: an approach using polymer expansions. (Russian)


The self-avoiding random walk in the weak sense, $w_\epsilon(T)$, differs from the simple symmetric random walk on the $d$-dimensional cubic lattice by giving small statistical weights tending to zero with $\epsilon \to 1$ to the trajectories intersecting themselves: $\epsilon = 0$ corresponds to the standard random walk, while $\epsilon = 1$ corresponds to the self-avoiding random walk in the strong sense. The effective small parameter is $\epsilon/d$. The following theorem is proved: If $d \geq 5$ and $\epsilon/d$ is sufficiently small then the mean square displacement of $w_\epsilon(T)$ shows diffusive behaviour and obeys the central limit theorem. The central limit theorem is formulated as follows: When diffusive rescaling is used, the characteristic function of the displacement tends to that of the $d$-dimensional standard Gaussian one. The proof is simpler and more natural than that of S. E. Golowich and J. Z. Imbrie [Ann. Physics 217 (1992), no. 1, 142–169; MR1173280]. It is based on statistical mechanical methods: polymer expansion for a special one-dimensional contour model (lace expansion).

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