Self-avoiding random walks in five or more dimensions: an approach using polymer expansions. (Russian)


The self-avoiding random walk in the weak sense, \( w_\varepsilon(T) \), differs from the simple symmetric random walk on the \( d \)-dimensional cubic lattice by giving small statistical weights tending to zero with \( \varepsilon \to 1 \) to the trajectories intersecting themselves: \( \varepsilon = 0 \) corresponds to the standard random walk, while \( \varepsilon = 1 \) corresponds to the self-avoiding random walk in the strong sense. The effective small parameter is \( \varepsilon/d \). The following theorem is proved: If \( d \geq 5 \) and \( \varepsilon/d \) is sufficiently small then the mean square displacement of \( w_\varepsilon(T) \) shows diffusive behaviour and obeys the central limit theorem. The central limit theorem is formulated as follows: When diffusive rescaling is used, the characteristic function of the displacement tends to that of the \( d \)-dimensional standard Gaussian one. The proof is simpler and more natural than that of S. E. Golowich and J. Z. Imbrie [Ann. Physics 217 (1992), no. 1, 142–169; MR1173280]. It is based on statistical mechanical methods: polymer expansion for a special one-dimensional contour model (lace expansion).

András Krámli