Action minimizing orbits in Hamiltonian systems.


This is a long and interesting survey on the dynamics of area-preserving mappings. The study of this subject dates from the pioneering work of Poincaré. He pointed out the intimate connection between the dynamics of area-preserving mappings and the dynamics of two-dimensional Hamiltonian systems. In 1981, Mather discovered a generalization of certain KAM invariant curves [in Mathematical analysis and applications, Part B, 531–562, Academic Press, New York, 1981; MR0634258]. Similar ideas were discovered by S. Aubry and his coworkers, independently, around the same time [see Regular and chaotic motions in dynamic systems (Erice, 1983), Plenum, New York, 1985; Zbl 661.70002]. Mather’s original approach was based on a variational principle introduced by I. C. Percival [J. Phys. A 7 (1974), 794–802; MR0363071]. The purpose of these lectures is to describe this generalization of the KAM invariant curves, the minimisation procedure used to prove their existence, and related constructions. The authors have made no attempt to survey the whole theory of Hamiltonian systems. Instead, their intention is to provide an introduction to the theory developed by Mather and Aubry, known as “Aubry-Mather” theory.

{For the collection containing this paper see MR1323219}