Loop condensation effects in the behavior of random walks.


This paper addresses two related problems on random walks. The first problem concerns the behavior of a random walk on \( \mathbb{Z}^d \) whose path depends upon the realization of a random field on the lattice. It is shown that if there is any interaction between the walk and the random field, then, for almost all realizations of the random field, the asymptotic behavior of the \( n \)-step probability distribution of the walk is not diffusive; that is, the distribution does not, after an appropriate scaling, converge to the Wiener measure.

The second problem concerns the intersection set of two independent simple random walks \( \omega' \) and \( \omega'' \) on \( \mathbb{Z}^d \), with \( d > 4 \). In particular, the question which is answered is how fast the probability that the intersection set contains \( k \) points decays with \( k \). This problem is phrased in two ways: let \( \xi = \# \{ (s \geq 0, \ t \geq 0) : \omega'(s) = \omega''(t) \} \) and \( \eta = \# \{ x \in \mathbb{Z}^d : \omega'(s) = \omega''(t) = x \ \text{for all} \ s \geq 0, \ t \geq 0 \} \). Then it is shown that, for all \( \delta > 0 \), there is a \( k_0(\delta) \) such that, for all \( k > k_0(\delta) \),

\[
(1 - 2/d + \delta) \exp(-k) \leq P(\xi \geq k) \leq (1 - 2/d - \delta) \exp(-k).
\]

It is further shown that there exist constants \( c_1, c_2 > 0 \) depending only on \( d \) such that, for all \( k \) large enough,

\[
\frac{1}{2} \exp(-c_1 k) \leq P(\eta \geq k) \leq \frac{1}{2} \exp(-c_2 k).
\]

The proofs of these results rely on a careful estimate of the probability that a simple random walk started at \( x \in \mathbb{Z}^d \) visits all points of the cube of side length \( 2k \) centered at \( x \) and estimates for the probability of an \( n \)-step loop in such a walk.

\{For the collection containing this paper see MR1311707\}

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