Let $g$ be the normalized “quadratic” Feigenbaum map of the interval $[-1, 1]$ (i.e.
$T(g)(x) := ag \circ g(\alpha^{-1}x) = g(x)$, $g(0) = 1$, $g(1) = -1/\alpha$, $g''(0) < 0$, $\alpha \approx 2.5029$). Let
$\Delta^n_0 := [-1/\alpha, 1/\alpha]$, $\Delta^n_k := g^k(\Delta^n_0)$, $k = 0, 1, \ldots, 2^n - 1$. The Feigenbaum attractor is
$F = \bigcap_n \bigcup_k \Delta^n_k$. Let $\mu(\Delta^n_k) = 2^{-n}$ denote the (only) $g$-invariant measure on $F$. The spectrum
is discrete: $\lambda_{n,r} = \exp \frac{2\pi ir}{2^n}$ for $r$ odd, and the corresponding eigenfunctions are
$e_{(n)}^{(r)}(x) = (e^{(n)}(x))^r$, where $e^{(n)}(x) = \exp \frac{2\pi ik2^{-n}}{x \in \Delta^n_k}$, $0 \leq k < 2^n$.

For any smooth real $f$ on $[-1, 1]$ with $\int f \, d\mu = 0$ one considers the Fourier coefficients:
$a^{(n)}_r := \int f e^{(n)}_r \, d\mu$. The authors express and study $S_n := \sum_r |a^{(n)}_r|^2$ with the help of
thermodynamic formalism quantities for the standard expanding map $\sigma$ preserving $F$
(which is the multiplication by $\alpha$ on the “half” of $F$ containing 0, and by $ag$ on the second half). The pressure $p(\beta)$ (free energy) of $-\beta U$ for $U \approx \log |\sigma'|$ and the Hausdorff
dimension of $F$ appear in the formulas.

The authors mention that for $h_\infty$, a limit of period-doubling bifurcations, if $T^n(h_\infty) \to g$ then $p(2)$ (appearing in $S_n$) is the same and is universal.


Part II contains proofs.

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