Exemples de flots hamiltoniens dont aucune perturbation en topologie $C^\infty$ n’a d’orbites périodiques sur un ouvert de surfaces d’égales. (French. English summary) [[Examples of Hamiltonian flows such that no $C^\infty$ perturbation has a periodic orbit on an open set of energy surfaces]]


Summary: “We propose to prove that the closing lemma is false for Hamiltonian vector fields on the torus $T^{2n+2}$, $n \geq 1$, in the $C^{k_0+1}$ topology, $k_0 > 2n + 1$, and for almost every constant symplectic form. There exist $H_0 \in C^\infty(T^{2n+2})$ and, for almost every constant symplectic form $w_A$ on $T^{2n+2}$, an open neighbourhood $U$ of $H_0$ in $C^{k_0+1}(T^{2n+2})$ such that, for $H \in U$, one has $[-\frac{1}{2}, \frac{1}{2}] \subset H(T^{2n+2})$, any $c \in [-\frac{1}{2}, \frac{1}{2}]$ is a regular value of $H$, and the Hamiltonian flow of $H$ for $w_A$ is $C^1$ conjugate on each component of $H^{-1}(c)$ to a linear Diophantine flow on $T^{2n+2}$. The proof considers similar examples to those found by E. Zehnder and uses the local theorem of Arnol’d and Moser of conjugacy of diffeomorphisms of tori to Diophantine translations.”

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