In a recent paper [Ann. Mat. Pura Appl. (4) 159 (1991), 229–254], the author introduced the following new definition: “Let $X$ be a $q$-complete space, $\dim X = n < +\infty$. An open subspace $Y$ of $X$ is said to be relatively $q$-complete in $X$ if for each compact subset $K \subseteq Y$, there is a $C^\infty$ strongly $q$-convex exhaustive function $\varphi_K$ on $X$ such that $K \subseteq \{ x \in X : \varphi_K(x) < 0 \} \subseteq Y$.” Under the above conditions, he proved that $Y$ is a $q$-Runge domain in $X$ (that is, the natural homomorphism $H^q(X, \Omega^p) \to H^q(Y, \Omega^p)$ has dense image for every $p = 0, 1, \ldots, n$, where $\Omega^p$ denotes the coherent sheaf on $X$ of singular holomorphic $p$-forms as defined by A. Ferrari [Ann. Scuola Norm. Sup. Pisa (3) 24 (1970), 65–77; MR0274810]). Moreover, in the same paper it was proved that one has $H_k(X, Y; G) = 0$ for $k > n + q$ and any abelian group $G$, and this new notion was used to prove several results about homology and cohomology with compact supports of $q$-convex spaces, avoiding the use of stratified Morse theory.

Note that $Y$ is an open 0-relatively 0-complete domain in the Stein space $X$ if and only if $Y$ is a Runge domain in $X$.

In the present paper this new notion is examined with regard to several standard constructions useful in the theory of complex spaces. In particular we recall the following property [see also V. Villani, ibid. (3) 20 (1966), 15–23; MR0201678; V. Ancona, Ann. Univ. Ferrara Sez. VII (N.S.) 20 (1975), 49–52; MR0387661]: Let $X$ be a $q$-complete space and let $E \to X$ be a holomorphic vector bundle on $X$. It is known that $E$ is a $q$-complete space. For each relatively $q$-complete open subspace $Y$ in $X$, $E|_Y$ is a relatively $q$-complete open subspace of $E$. Finally, we also mention the following result. Let $X, Y$ be complex spaces and let $f : X \to Y$ be a finite holomorphic map. Let us assume that $Y$ is a $q$-complete space and that $Z$ is an open relatively $q$-complete subspace in $Y$. Then $f^{-1}(Z)$ is an open relatively $q$-complete subspace in $X$.

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