The Gaussian distribution for sums of distances. (Russian)


From the text (translated from the Russian): “Into a metric space with a probability measure we independently throw $n$ points. Let $p_n$ denote the sum of the $\binom{n}{2}$ pairwise distances, and let $
abla_n = (\rho_n - \mathbf{E}\rho_n)(D\rho_n)^{-1/2}$. We calculate the limit distribution $\Phi$ of the sequence $\nabla_n$ and obtain a simple test for its normality. A space is said to be normal if the distribution $\Phi$ is normal, and homogeneous if the mean distance from any given point to the remaining points is the same for almost all points. Spheres and their products, in particular, tori with a ‘natural’ measure and metric, are homogeneous spaces.

“We show that the absence of homogeneity is a sufficient and ‘almost’ necessary condition for normality. Thus, normality is a typical property and nonnormality is exceptional. However, exceptions—homogeneous spaces—are very important. We calculate the limit distribution $\Phi$ in this case as well.

“Pairwise distances do not form systems of independent random variables and, therefore, the central limit theorem is inapplicable. However, two distances are independent if all four points are pairwise distinct. The results obtained can be considered as a limit theorem for such ‘weakly dependent’ random variables. We formulate sufficient conditions for independence for a set of distances in Section 5.”

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