Homeomorphisms of the circle with fracture-type singularities. (Russian)

"Fracture-type singularities" means that the homeomorphisms under consideration have a jump in the first derivative. More precisely, the authors consider a family of monotone mappings \( f_\Omega(x) = f(x) + \Omega \) with \( f(x) \) satisfying the following conditions: (a) \( f(x + 1) = f(x) + 1 \); (b) there is a unique \( x_0 \in [0, 1) \) such that \( f'_-(x_0) \neq f'_+(x_0) \) (the + and − indicate directional derivatives); (c) \( f(x) \in C^{2+\epsilon}([0, x_0) \cup (x_0, 1]) \), \( f'(x) > \text{const} > 0 \), \( x \neq x_0 \) (misprinted in the paper), and the corresponding family \( T_{f_\Omega}(x) \) of homeomorphisms of the circle \( T_{f_\Omega}(x) = \{f_\Omega(x)\} \). There are three fairly technical assertions (due to Khanin), two relating to the convergents to the (infinite) continued fraction expansion of an irrational rotation number. The third result is stated simply as follows: The set of values of the parameter that correspond to irrational rotation numbers has Lebesgue measure zero. Some computational results (due to Vul) are also included. 

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