A saddle connection is another name for a singular trajectory. It is an arc associated with a nonzero holomorphic quadratic differential \( q(z) \, dz^2 \) on a closed Riemann surface \( X \) of genus \( > 1 \) and satisfies the following two conditions: (i) \( \arg q(z) \, dz^2 \) is constant along the arc; (ii) the two endpoints are zeroes of \( q \), while there are no zeroes of \( q \) in its interior. Thus a saddle connection is a geodesic for the metric \( |\sqrt{q} \, dz| \) which joins two zeroes of \( q \) and contains no zeroes in the interior. The quadratic differential \( q \) is supposed to be normalized so that \( \int_X |q| = 1 \). The author shows that the asymptotic growth rate of the number of saddle connections of length \( \leq T \) does not depend on the genus and is at most \( T^2 \). More precisely, if we denote by \( N_1(T) \) the number of parallel families of closed regular trajectories of length \( \leq T \) and by \( N_2(T) \) the number of saddle connections of length \( \leq T \), then \( \limsup_{T \to \infty} (N_1(T)/T^2) \leq \limsup_{T \to \infty} (N_2(T)/T^2) < \infty \). As an application of this theorem to the study of billiards the author also proves the following theorem: Let \( \Delta \) be a rational billiard table, and let \( D_T(\Delta) \) be the number of generalized diagonals of length \( \leq T \). Then \( \limsup_{T \to \infty} (D_T(\Delta)/T^2) < \infty \).

The basic tool for the proof of these theorems is the Teichmüller map; it is used to construct a sequence of Riemann surfaces and systems of saddle connections. The construction is technically rather difficult and a number of new notions basic for counting saddle connections also appear in the paper. Nevertheless, the paper is thoroughly understandable, since the author often explains his ideas, plan and motivation and illustrates the theorems with examples.

Masakazu Shiba